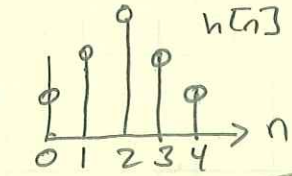
4 Types of FIR filters

	N	length h[n]	Type
Symmetric around N/2	even	odd	I
	odd	even	II
Antisymmetric around N/2	even	odd	III
	odd	even	IV

Type I

ex :



$N=4$

example

$$H(z) = h[0] + \overbrace{h[1]z^{-1} + \overbrace{h[2]z^{-2} + h[3]z^{-3}}^{\text{same}} + h[4]z^{-4}}$$

$$= h[0](z^0 + z^{-4}) + h[1](z^{-1} + z^{-3}) + h[2]z^{-2}$$

$$= \underset{\substack{\uparrow \\ \text{delay } 2}}{z^{-2}} \Big[\underbrace{h[0](z^2+z^{-2}) + h[1](z^1+z^{-1}) + h[2]}_{}\Big]$$

must be zero phase = real (proof below)

Proof (eg) $(z^2 + z^{-2}) \Leftrightarrow (e^{j\omega 2} + e^{-j\omega 2})$
$= 2\cos(2\omega) \leftarrow$ real

(eg) $(z^1 + z^{-1}) \Leftrightarrow e^{j\omega} + e^{-j\omega}$
$= 2\cos(\omega) \leftarrow$ real

so,

$$|H(e^{j\omega})| = \boxed{h[0]\cos(2\omega) + h[1]\cos(\omega) + h[2]}$$

$$\theta(\omega) = \angle e^{-j2\omega} + \angle(\text{real number}) \rightarrow 0$$

$$= \boxed{-2\omega}$$

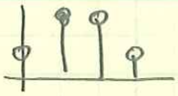
Group delay $\tau_g(\omega) = -\frac{d\theta}{d\omega} = \boxed{2 \text{ sample delay}}$

For any N

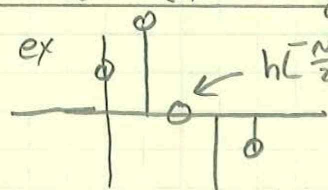
- $H(e^{j\omega}) = \underbrace{e^{-j\frac{N}{2}\omega}}_{\text{delay}} \underbrace{\left[2\sum_{n=1}^{N/2} h[\tfrac{N}{2}-n]\cos(\omega n) + h[\tfrac{N}{2}]\right]}_{\text{zero phase FIR filter}}$

  $h[\frac{N}{2}] \uparrow$ center value

- $h[n] = h[N-n]$ (periodic symmetric)
- $\theta(\omega) = -(\frac{N}{2}\omega)$
- $\tau_g = \frac{N}{2}$

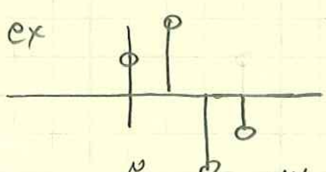
Type II ex  like Type I but with no single middle value

- $H(e^{j\omega}) = \underbrace{e^{-j\frac{N}{2}\omega}}_{\text{non integer delay}} \left[2 \sum_{n=1}^{\frac{N+1}{2}} h\left[\frac{N+1}{2} - n\right] \cos\left(\omega\left(n - \frac{1}{2}\right)\right) \right]$
- $h[n] = h[N-n]$ (per symm)
- $\theta(\omega) = -\frac{N}{2}\omega$
- $\tau_g = -\frac{N}{2}$ (non integer!)

Type III ex  $h[\frac{N}{2}]$ must be zero always

end up with $e^{j\omega} - e^{-j\omega}$ => sin's

- $H(e^{j\omega}) = j\, e^{-j\frac{N}{2}\omega} \left[2 \sum_{n=1}^{N/2} h\left[\frac{N}{2} - n\right] \sin(\omega n) \right]$
- $h[n] = -h[N-n]$ periodic antisymm
- $\theta(\omega) = -\frac{N}{2}\omega$
- $\tau_g = -\frac{N}{2}$

Type IV ex 

- $H(e^{j\omega}) = e^{-j\frac{N}{2}\omega} \left[2 \sum_{n=1}^{\frac{N+1}{2}} h\left[\frac{N+1}{2} - n\right] \sin\left[\omega\left(n - \frac{1}{2}\right)\right] \right]$
- $h[n] = -h[N-n]$ per. antisymm
- $\theta(\omega) = -\frac{N}{2}\omega$
- $\tau_g = -\frac{N}{2}$ (non-integer)