Objectives

- Transfer Function Review
  - How to find it ✓
  - 4 Types of Notation of $H(e^{j\omega})$ ✓
  - Poles, zeros & MATLAB ✓
  - Find freq. response given $H(z)$ ✓
- How to find $y[n]$ given $x[n]$, $h[n]$
- Geometrical interpretation $H(z)$
- BIBO stability

Find $y[n]$ given $x[n]$, $h[n]$

- z transforms $Y(z) = X(z)\,H(z)$

  $y\cancel{[n] = x[n]} \cdot h[n]$

- Convolve $y[n] = x[n] * h[n]$
- recast $x[n]$ as $e^{j\omega_0 n}$

  $y_{ss}[n] = x[n] \cdot H(e^{j\omega_0})$

  $y[n] = y_{ss}[n] + \cancel{y_{transient}}[n]$ 0 for $n \geq M$

Transfer Function

How to find $H(e^{j\omega})$ given $h[n]$
- $\to$ FIR  $h[n] = \delta[n] - \delta[n-1]$ $\quad H(z) = 1 - z^{-1}$
- $\to$ IIR $u[n]$ $\quad H(z) = \frac{1}{1 - z^{-1}}$

- $h[n] \longrightarrow H(z)$ tables $\longrightarrow H(e^{j\omega})\Big|_{z = e^{j\omega}}$

- D.E. $x[n] - \frac{1}{2}x[n-1] = y[n] + \frac{3}{4}y[n-2]$

  $X(z)\left[1 - \frac{1}{2}z^{-1}\right] = Y(z)\left[1 + \frac{3}{4}z^{-2}\right]$

  $H(z) = \frac{Y}{X} = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-2}} \longrightarrow H(e^{j\omega})\Big|_{z = e^{j\omega}}$

- Block Diagram
  - z transform
  - Variables output of ⊕
  - Eqn summing around ⊕
  - $X(z)$ $\quad Y(z) \Rightarrow H(z)$

- Filter Specification Handout last class $N = 0$, $M = 2$ 3 coef.

$\cos(\omega_0 n) \longrightarrow \boxed{H(e^{j\omega})} \longrightarrow A\cos(\omega_0 n + \theta)$

$A\angle\theta = H(e^{j\omega_0})$

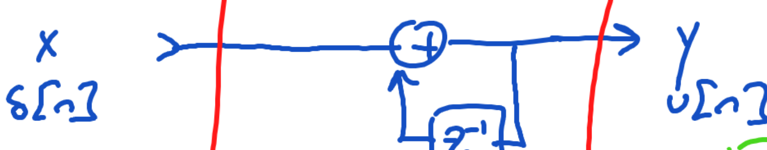
BIBO Stability

$x[n] \longrightarrow \boxed{H(z)} \longrightarrow y[n]$

input is bounded ⟶ bounded output

B.I. ⟶ B.O.

$h[n] = u[n]$

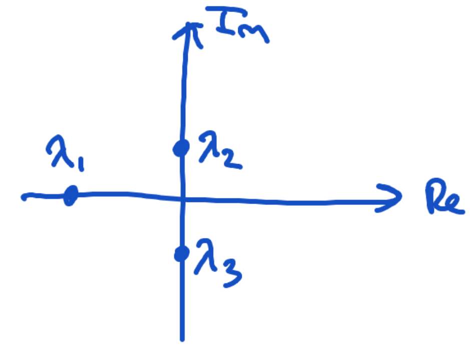


$\boxed{u[n]} \longrightarrow H(z) = \frac{1}{1-z^{-1}} \longrightarrow \boxed{n\,u[n]}$

Is it BIBO Stable?

- $\sum_{n=-\infty}^{\infty} |h[n]| < \infty \Rightarrow$ BIBO Stable
- Are $|\text{poles}| < 1 \Rightarrow$ BIBO Stable

$$H(z) = \frac{\text{num}}{(1-\lambda_1 z^{-1})(1-\lambda_2 z^{-1})(\cdots)}$$



$$Y(z) = X(z)H(z)$$

$$y[n] = (\lambda_1)^n + (\lambda_2)^n + (\lambda_3)^n + \dots$$