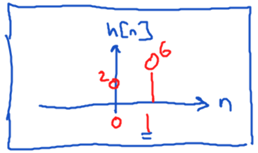
Example

$x[n] = \left(\frac{1}{2}\right)^n u[n] \longrightarrow$



$\longrightarrow y[n] = y_{transient}[n] + y_{ss}[n]$

$H(z)$ FIR: $M=1$, $N=0$, den $H(z)=1$, $y[n]$ = current + past inputs

Find $y[n]$

Method 1 Exact z transforms

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$H(z) = 2 + 6z^{-1}$$

$$Y(z) = X(z)H(z)$$

$$= \frac{2+6z^{-1}}{1-\frac{1}{2}z^{-1}} \quad M=1,\ N=1$$

$$\boxed{-12 + \frac{14}{1-\frac{1}{2}z^{-1}}}$$

$$-\tfrac{1}{2}z^{-1}+1 \,\big)\, 6z^{-1} + 2$$
$$\mp 6z^{-1} \pm 12$$
$$14$$

$$Y(z) = -12 + \frac{14}{1-\frac{1}{2}z^{-1}}$$

$$\boxed{y[n] = -12\,\delta[n] + 14\left(\tfrac{1}{2}\right)^n u[n]}$$

Method 2

$$\ln\left(\left(\tfrac{1}{2}\right)^n\right) = \ln\left(e^{j\omega_0 n}\right)$$

$$\cancel{n}\ln\left(\tfrac{1}{2}\right) = j\omega_0 \cancel{n}$$

$$\omega_0 = \frac{\ln\left(\frac{1}{2}\right)}{j}$$

$e^{j\omega_0 n} \longrightarrow \boxed{H(z)} \longrightarrow H(e^{j\omega_0})\, e^{j\omega_0 n}$

gain complex # — same function input

$$\boxed{\omega_0 = \frac{\ln\left(\frac{1}{2}\right)}{j}}$$

$$H(z) = 2 + 6z^{-1}$$

$$\boxed{H(e^{j\omega}) = 2 + 6e^{-j\omega}}$$

$$y[n] = y_{transient}[n] + y_{ss}[n]$$
$$+ \left(\tfrac{1}{2}\right)^n u[n]\, H(e^{j\omega_0})$$
$$+ \left(\tfrac{1}{2}\right)^n u[n]\left(2 + 6e^{-\cancel{j}\frac{\ln(\frac{1}{2})}{\cancel{j}}}\right)$$
$$+ \left(\tfrac{1}{2}\right)^n u[n]\left(2 + 6\cdot 2\right)$$
$$\underbrace{y_{trans}[n]}_{0 \le n < M} \quad + 14\left(\tfrac{1}{2}\right)^n u[n]$$

$0 \le n < 1$

$n = 0$

$$\boxed{n > 0,\ y[n] = 14\left(\tfrac{1}{2}\right)^n u[n]}$$

$$\left(e^{-\ln\left(\frac{1}{2}\right)}\right) = \left[e^{\ln\left(\frac{1}{2}\right)}\right]^{-1}$$
$$= \left(\tfrac{1}{2}\right)^{-1}$$
$$= 2$$

$$\ln\left(\tfrac{1}{2}\right) = \ln(1) - \ln(2)$$
$$= 0 - \ln(2)$$

$$e^{\ln(2)} = 2$$