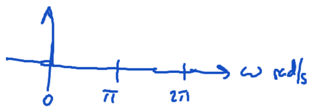
DTFT

DFT

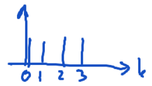
Z transform

$\angle X(e^{j\omega})$ rad

$|X(e^{j\omega})|$



$|X[k]|$



	DTFT $X(e^{j\omega})$	DFT $X[k]$	Z
Forward	$\sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$	$\sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi k}{N}}$	$\sum_{n=-\infty}^{\infty} x[n] z^{-n}$
Inverse	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$ rarely! (Tables)	$\frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{\frac{j2\pi k}{N}}$	Don't do! → $\oint$ (Tables)
Ex. Finite Length [2 6 3] ↑	$2+6e^{-j\omega}+3e^{-j\omega 2}$	← Sample at $\omega = \frac{2\pi k}{N}$ for $k = 0, 1, 2$; $X[k=0] = 2+6+3 = 11$	$2+6z^{-1}+3z^{-2}$ ROC $\|z\| > 0$

DTFT (cont. freq, ∞ length)

- exists if $x[n]$ has finite energy $\mathcal{E}_x \equiv \sum |x[n]|^2 < \infty$ ex $(\frac{1}{2})^n u[n]$ ✓, $2^n u[n]$ ✗
- Periodic in 2π, Conj symm. around π
- Confusion: $\omega = 0 \ldots \pi \ldots 2\pi$ rad/sample but $\angle X(e^{j\omega})$ measured in rads
  - 0 ↑ DC, π ↑ HF, 2π ↑ DC
- Energy density spectrum $S_{xx}(e^{j\omega}) = |X(e^{j\omega})|^2$, $\frac{1}{2\pi}\int_{-\pi}^{\pi} S_{xx}(e^{j\omega}) d\omega = \mathcal{E}_x$
- Tables, properties, symmetries CS $x[n]$ → Re $X(e^{j\omega})$; CA $x[n]$ → Im "
- Graph using MATLAB: X = freqz(num, den, w) plot(w, abs(X))

DFT (discrete, finite length)

- if $x[n]$ length N, so is $X[k]$. Both $x[n]$, $X[k]$ $0 \ldots N-1$
- Sampled version of DTFT $\omega = \frac{2\pi k}{N}$
- Calc. using FFT algorithm
- fft(x, N) — N length of DFT, zero pad x to make it N long
- Tables, properties, symmetries
  - pcs → Re DFT
  - pca → Im DFT
- Circ. conv. freq.-aliased linear convolution
  - $x[n]$, $h[n]$, both length N
  - $x \circledast h$, length N
  - linear convolution
  - $x * h$, length $2N-1$
  - IDFT(DFT(padded x) • DFT(padded h))
  - in MATLAB $x[n]$ length N_x, $h[n]$ length N_h
  - >> $N = N_x + N_h - 1$
  - >> y = ifft(fft(x, N) .* fft(h, N))

Z transform

- Tables, properties, symmetries

  $x[n]$ is CS $\rightarrow X(z)$ real

  " CA " imag

$$z^{(\cdot)} \frac{(1-\xi_1 z^{-1})(1-\xi_2 z^{-1})\cdots}{(1-\lambda_1 z^{-1})(1-\lambda_2 z^{-1})\cdots} = z^{(\cdot)}\left[\frac{\quad}{1-\lambda_1 z^{-1}} + \frac{\quad}{1-\lambda z^{-1}}\right]$$

- Inverse Z transform

	analytic (formula)	numeric
hand	PFD, tables	long division
MATLAB	residuez	filter(num, den, input) input = [1 0 0 0 ...]

Ex

$$X(z) = \frac{7.5 - 25z^{-1} + z^{-2} + 42z^{-3}}{1 - 5z^{-1} + 6z^{-2}} \qquad M = 3 \quad N = 2$$

[r, p, k] = residuez([7.5 -25 1 42], [1 -5 +6])

r = [2.5 -1]

p = [3 2]

k = [6 7]

$$6 + 7z^{-1} + \frac{2.5}{1-3z^{-1}} + \frac{-1}{1-2z^{-1}}$$

x = filter([7.5 -25 ...] (num), [1 -5 6] (den), [1 zeros(1, 100)])