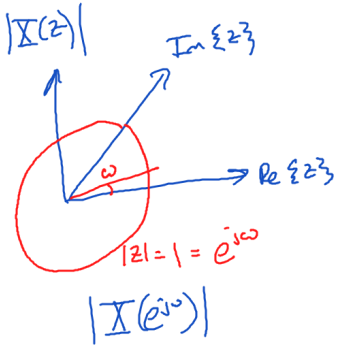
$x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

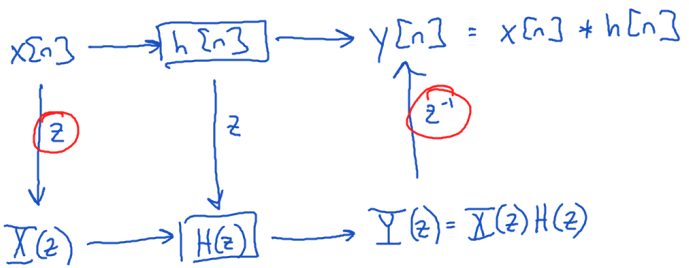


$$x[n] = [1 \quad 2 \quad 1] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$

$$X(z) = 1 + 2z^{-1} + z^{-2}, \quad |z| > 0$$

$$= 1 + \frac{2}{z} + \frac{1}{z^2}$$

Why Z transforms?



z^{-1} can proper

$$\frac{8 - z^{-1}\cdot 0^{M}}{1 - \frac{1}{2}z^{-1}\cdot 1^{N}}$$

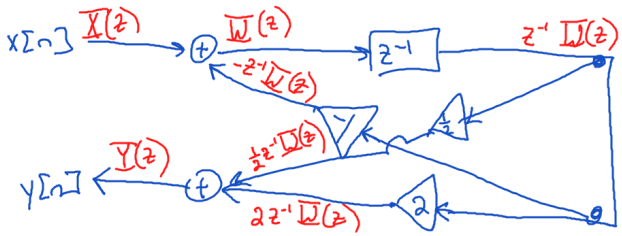
want $N > M$ $\Rightarrow$ proper

$$= 2 + \frac{6}{1 - \frac{1}{2}z^{-1}}$$

$$\Leftrightarrow \boxed{2\delta[n] + 6\left(\tfrac{1}{2}\right)^{n} u[n]}$$

$$\begin{array}{r|l} & 2 + \frac{6}{1-\frac{1}{2}z^{-1}} \\ -\frac{1}{2}z^{-1} + 1 & -z^{-1} + 8 \\ & -z^{-1} + 2 \\ \hline & 6 \end{array}$$

Example Block Diagram → $H(z)$



Analyze BD
1) Create new variable at output of every summer
2) Write eqns around each summer
3) Simplify by eliminating intermediate eqns

Top ⊕ : $W(z) = X(z) - z^{-1} W(z) \Rightarrow W(1+z^{-1}) = X(z) \Rightarrow W(z) = \frac{1}{1+z^{-1}} X(z)$

Bot ⊕ : $Y(z) = \frac{1}{2} z^{-1} W(z) + 2 z^{-1} W(z) \Rightarrow Y(z) = \frac{5}{2} z^{-1} W(z)$

$$Y(z) = \frac{5}{2} z^{-1} \cdot \frac{1}{1+z^{-1}} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \boxed{\frac{5}{2} z^{-1} \cdot \frac{1}{1+z^{-1}}}$$

Find $h[n]$

$$H(z) = \left(\frac{5}{2} z^{-1}\right)\left(\frac{1}{1+z^{-1}}\right)$$

$$a^n u[n] \Leftrightarrow \frac{1}{1-az^{-1}}$$

$$\boxed{h[n] = \frac{5}{2}(-1)^{n-1} u[n-1]}$$

Find $y[n]$ if input $x[n] = 2\left(\frac{1}{2}\right)^n u[n]$

$$Y(z) = H(z) X(z)$$

$$= \left(\frac{5}{2} z^{-1}\right)\left(\frac{1}{1+z^{-1}}\right)\left(2\right)\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

$$= \left(5 z^{-1}\right)\left(\frac{1}{1+z^{-1}}\right)\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

$$= \left(5z^{-1}\right)\left[\frac{\frac{1}{1-\frac{1}{2}} = \frac{2}{3}}{1+z^{-1}} + \frac{\frac{1}{1+2} = \frac{1}{3}}{1-\frac{1}{2}z^{-1}}\right]$$

$$= \left(5z^{-1}\right)\left(\frac{2/3}{1+z^{-1}}\right) + \left(5z^{-1}\right)\left(\frac{1/3}{1-\frac{1}{2}z^{-1}}\right)$$

$$= \left(\frac{10}{3} z^{-1}\right)\left(\frac{1}{1+z^{-1}}\right) + \left(\frac{5}{3} z^{-1}\right)\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)$$

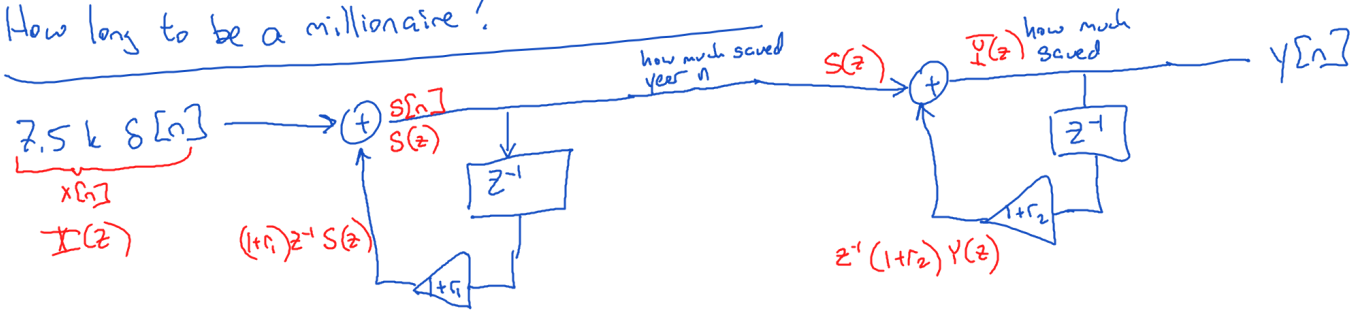
$$\boxed{y[n] = \frac{10}{3}(-1)^{n-1} u[n-1] + \frac{5}{3}\left(\frac{1}{2}\right)^{n-1} u[n-1]}$$

Say your first-job 50k and save 15% of that $\Rightarrow$ 7.5k

Every year you get a 2.5% raise r_1

On average investments earn 7.5% in interest r_2

How long to be a millionaire?



(1) Write new variable output of every summer ✓

(2) Write an eqn around each summer ✓

left (+): $S(z) = X(z) + (1+r_1) z^{-1} S(z)$

- $S(z)[1 - (1+r_1) z^{-1}] = X(z)$

right (+): $Y(z) = S(z) + z^{-1}(1+r_2) Y(z)$

- $Y(z)[1 - z^{-1}(1+r_2)] = S(z)$

$$Y(z)[1 - z^{-1}(1+r_2)][1 - (1+r_1) z^{-1}] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{[1 - z^{-1}(1+r_2)][1 - z^{-1}(1+r_1)]}$$

$$Y(z) = H(z) X(z)$$

$$= \frac{7.5k}{(1 - 1.075 z^{-1})(1 - 1.025 z^{-1})}$$

$$= \frac{\frac{7.5k}{1 - 1.025(\frac{1}{1.075})}}{1 - 1.075 z^{-1}} + \frac{k_1 = -153.75k}{1 - 1.025 z^{-1}}$$

$k_2 = 161.25k$

$$y[n] = k(1+r_1)^n + k(1+r_2)^n$$

$$= [(-153.75k)(1.025)^n + (161.25k)(1.075)^n]\, u[n]$$

$$y[29] = 1.031k$$