Obj Z transforms
- Big picture ✓
- Math ✓
- Tables ✓

Z vs DTFT ✓

Notation

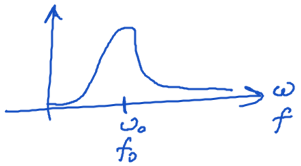
Graphing Z

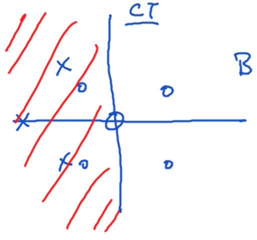
Big Picture

CT	DT	Use
Fourier Transform $X(j\omega)$ = Bode Plot $H(j100) = A\angle\theta$ $\cos(100t) \rightarrow \boxed{H(j\omega)} \rightarrow A\cos(100t+\theta)$	$\Leftrightarrow$ DTFT $X(e^{j\omega})$ $\lvert X(e^{j\omega})\rvert$ vs ω	• evaluate freq-dependent energy content
Laplace Transforms $X(s)$ $\int x(t)e^{-st}\,dt$	$\Leftrightarrow$ Z transform $X(z)$	• analytically solve y given x, h • determine BIBO stability $= \sum \lvert h[n]\rvert$ $\quad \int \lvert h(t)\rvert < \infty$ CT = all poles LHP

$X(e^{j\omega})$ $H(e^{j\omega})$

$X(j\omega)$ $H(j\omega)$





BIBO stable ✓

Math

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}}$$

Complex #

notes

- for $\sum_{n=0}^{\infty} a^n$ to exist, $|a| < 1$
- z is complex (like s)
- z transform exists for more sequences than DTFT does

ex $2^n u[n] \Rightarrow X(e^{j\omega}) = \sum_{n=0}^{\infty} 2^n e^{-j\omega n}$ blows up

$$X(z) = \sum_{n=0}^{\infty} 2^n z^{-n} = \boxed{\sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n} \qquad z^{-1} = \frac{1}{z}$$

as $\left|\frac{2}{z}\right| < 1 \qquad \frac{|2|}{|z|} < 1 \qquad |2| < |z| \qquad \boxed{|z| > 2}$

$$= \boxed{\frac{1}{1 - 2z^{-1}}, \quad |z| > 2}$$

Z trans region of convergence ROC

• General: $\boxed{a^n u[n] \Leftrightarrow \frac{1}{1 - a z^{-1}}, \quad |z| > a}$

Table

Recall

$$y = \sum_{n=0}^{\infty} a^n$$

$$y = 1 + a + a^2 + a^3 + \cdots$$

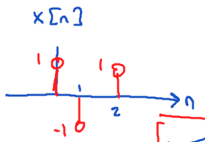
$$ay = a + a^2 + a^3 + a^4 + \cdots \qquad \text{Sub}$$

$$y - ay = 1$$

$$y(1-a) = 1$$

$$y = \frac{1}{1-a}, \quad |a| < 1$$

Ex



$$= \delta[n] - \delta[n-1] + \delta[n-2]$$

$$\boxed{X(z) = 1 - z^{-1} + z^{-2}}, \quad \text{all } z > 0$$

Z vs DTFT

$X(z) =$

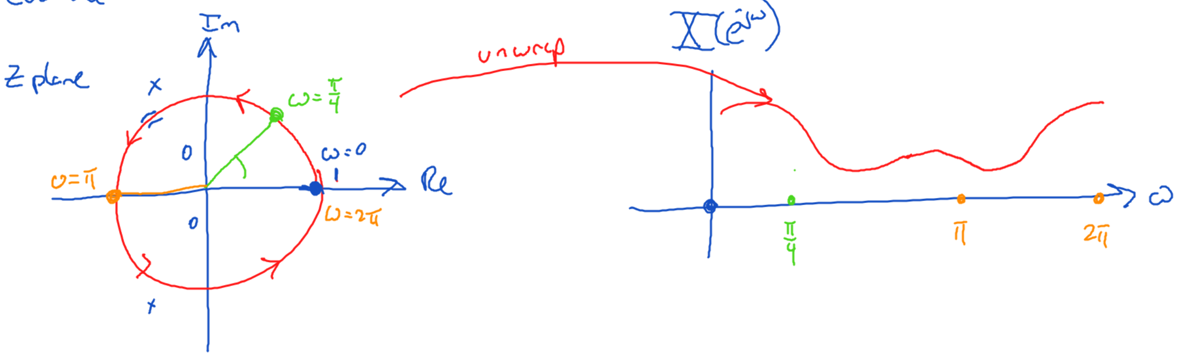
$\sum x[n] z^{-n}$

$X(e^{j\omega}) =$

$\sum x[n] e^{-j\omega n}$

$z \Leftrightarrow e^{j\omega}$

evaluate $z = e^{j\omega}$ (around unit circle) to find DTFT



$|X(z)|$

$u[n] \Leftrightarrow \frac{1}{1-z^{-1}}$, $|z| > 1$

poles? $z^{-1} = 1$ $\quad$ $z = 1$ pole

$u[n] \Leftrightarrow \frac{z}{z-1}$ $\quad$ $\frac{z=0 \text{ zero}}{z=1 \text{ pole}}$

$|X(z)|$

DTFT $\quad u[n] \Leftrightarrow \frac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k)$

Time Delay

Inverse transform of

$$X(z) = \frac{z^2}{1+3z^{-1}} + \frac{z^{-3}}{1-z^{-1}}, \qquad |z| > 3$$

$$= z^2 \underbrace{\left(\frac{1}{1+3z^{-1}}\right)}_{(-3)^{n+2} u[n+2]} + z^{-3} \underbrace{\left(\frac{1}{1-z^{-1}}\right)}_{\cancel{1^n}\, 1\, u[n-3]}$$

$$= \boxed{(-3)^{n+2} u[n+2] + u[n-3]}$$

$$= \boxed{9(-3)^n u[n+2] + u[n-3]}$$

Aside

$$a^n u[n] \Leftrightarrow \frac{1}{1-az^{-1}}$$

$$x[n] \Leftrightarrow X(z)$$

$$x[n-n_0] \Leftrightarrow X(z)\, z^{-n_0}$$