Obj

DFT properties

- circular shift
- circular convolution
- linear convolution using DFTs

Review

Delay in time do to a DTFT

$x[n] \Leftrightarrow X(e^{j\omega})$ | $x[n-2] \Leftrightarrow X(e^{j\omega})\, e^{-j2\omega}$

(Delay?) for DFT circular shift by 1

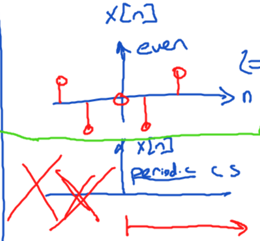
$x[n] = [1\ 2\ 3\ 4]$ | $[4\ 1\ 2\ 3]$

$\Leftrightarrow X[k]$ | $\Leftrightarrow X[k]\, e^{-j\frac{2\pi k}{N}(-1)}$

In general,

$$\boxed{x[\langle n-n_0\rangle_N] \Leftrightarrow X[k]\, e^{-j\frac{2\pi k}{N} n_0}}$$

Review



DTFT $X(e^{j\omega})$

$\Leftrightarrow$ real

DFT finite length, begins at $n=0$

real

Circular Convolution of 2 length-N sequences

math - regular convolution

$\underbrace{x[n]}_{N} * \underbrace{h[n]}_{N} = \sum_{k=0}^{N-1} x[k]\, h[n-k]$ (length $2N-1$)

math - circular convolution

- $x[n] \circledast h[n] = \sum_{k=0}^{N-1} x[k]\, h[\langle n-k \rangle_N]$ (length N)
- intuitively - graphically convolve x, h then time-domain alias to make length N

ex $x[n] = [1 \quad 1 \quad 1] \ 0 \ 0$

$h[n] = [1 \quad 1 \quad 1] \ 0 \ 0$

$y[n] = x[n] * h[n] = [1 \quad 2 \quad 3 \quad 2 \quad 1]$

$x[n] \circledast h[n] = [\underset{1+2}{3} \quad \underset{2+1}{3} \quad \underset{3}{3}]$ - - - - -

- Computationally IDFT(DFT(x) · DFT(h))

ex
```
>> x = [1 1 1]
>> h = [1 1 1]
>> ifft(fft(x) .* fft(h))
>> [3 3 3]
```

Linear Convolution using DFT's

zero pad x, h to make them $N_x + N_h - 1$ long!

$x1 = [x \ \ \text{zeros}(1, N_h - 1)]$

$h1 = [h \ \ \text{zeros}(1, N_x - 1)]$

$y1 = \text{ifft}(\text{fft}(x1) .* \text{fft}(h1))$ length $N_x + N_h - 1$

So what?

circ conv. is bad.

but easy to perform using fft

[zero pad we get usefulness of linear convolution w/ speed of using fft