DFT - Discrete Fourier Transform

(not DTFT " Time " ")

- Intuition
- Math
- Matlab

DTFT - hard to compute N^2

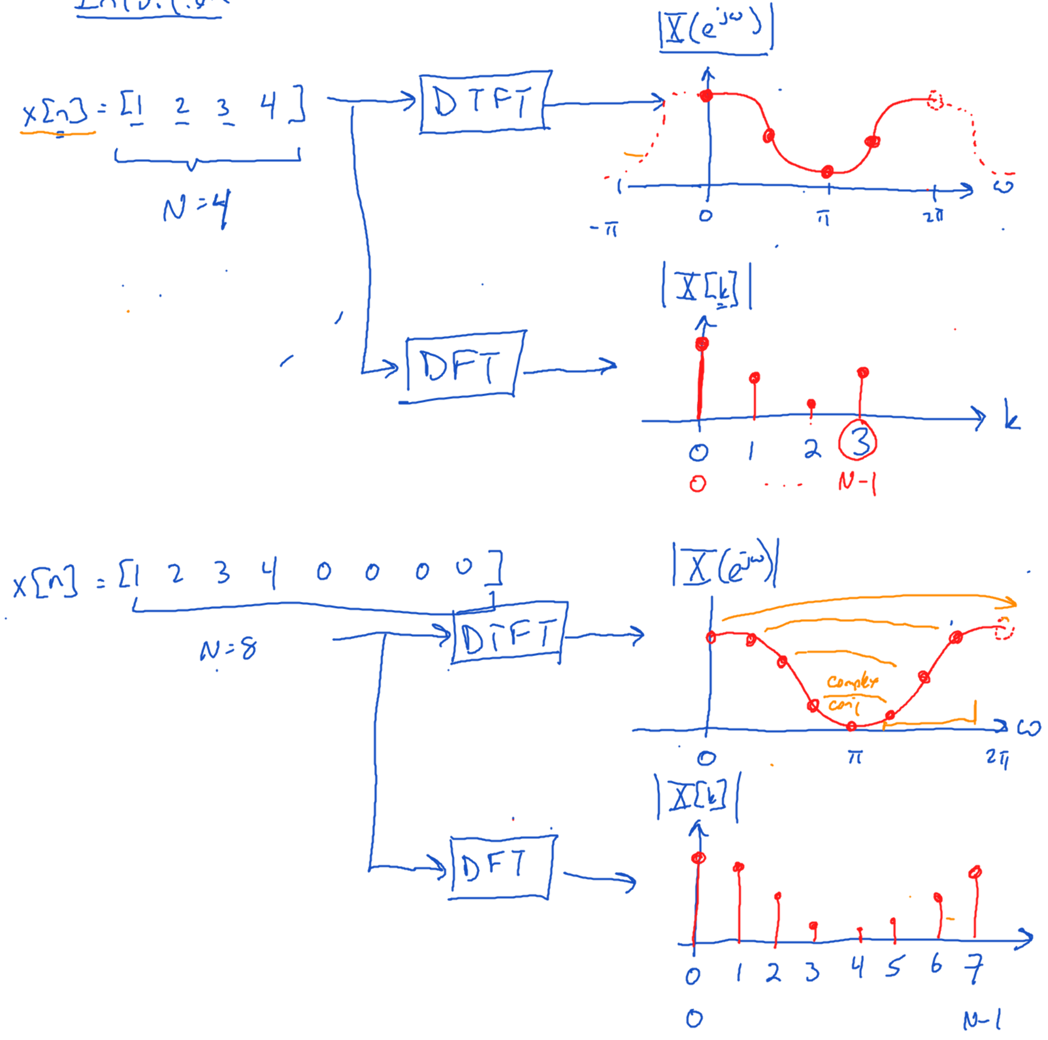
DFT - easy to compute $N \log_2 N$

FFT fast Fourier Transform

DFT

- sequence in → sequence out
  ⇒ Perfect for MATLAB!
- $x[n]$, $X[k]$ both: finite length N; n, k: $0 \dots N-1$
- $x[n] \longleftrightarrow X[k]$

Intuition



Math

$$\underset{\text{DFT}}{X[k]} = \underset{\text{DTFT}}{X(e^{j\omega})}\Big|_{\omega = \frac{2\pi k}{N}} = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}}, \quad 0 \le k \le N-1$$

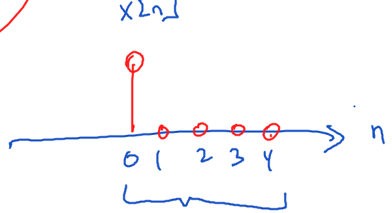
N: # of samples in $x[n]$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi k n}{N}}$$

Ex

finite-length sequence $N=5$

$$x[n] = \begin{cases} 1, & n=0 \\ 0, & \text{ow} \end{cases}$$



$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}}$$

$$\boxed{x[0]e^{0}} + \cancel{x[1]}\, e^{-j\frac{2\pi k}{N}} + \cancel{x[2]}\, e^{-j\frac{2\pi 2k}{N}} + \dots \cancel{x[N-1]}\, e^{-j\frac{2\pi k(N-1)}{N}}$$

$$= \sum_{n=0}^{\boxed{0}} 1 \cdot e^{-j\frac{2\pi k n}{N}}$$

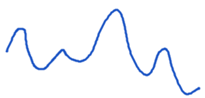
$$= e^{-j2\pi k\frac{0}{5}}$$

$$= e^{0}$$

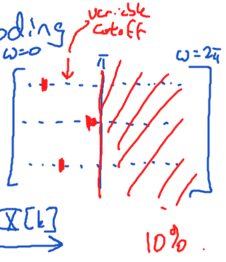
$$= 1$$

$$= [1\ \ 1\ \ 1\ \ 1\ \ 1]$$

Real-World Ex MP3 encoding



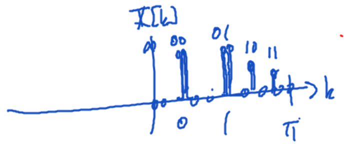
$$\begin{bmatrix} 6 & -2 & 3 & 8 & 2 & -14 & 6 \\ 9 & 2 & 3 & -4 & -5 & 6 & 2 \\ -4 & 6 & 8 & -2 & -2 & 3 & 8 \end{bmatrix}$$

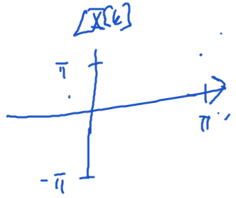


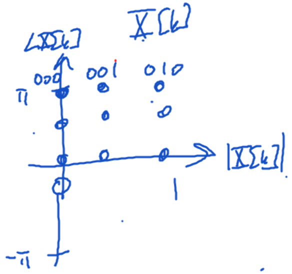
$x(t) \longrightarrow$ A/D $\xrightarrow{x[n]}$ DFT $\xrightarrow{X[k]}$

$f_s = 44.1$ kHz

$$X[k] = |X[k]|\ \angle X[k]$$







Matlab

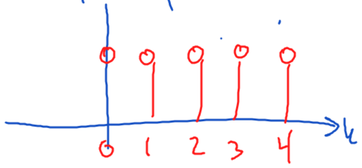
$$\boxed{\begin{array}{l} \texttt{>> } X = \texttt{fft}(x); \\ \texttt{>> } x = \texttt{ifft}(X); \end{array}}$$

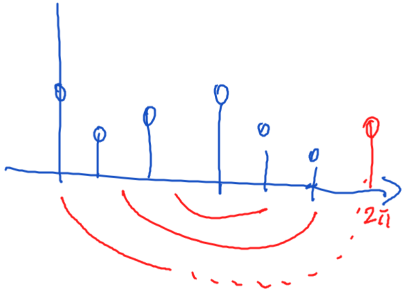
ex

```
>> x = zeros(1, 5);
>> x(1) = 1;
>> X = fft(x);
>> k = 0:4;
>> stem(k, abs(X))
```

$x = [1 \quad 0 \quad 0 \quad 0 \quad 0]$

$|X[k]|$





$$x[n] = [1 \quad 2]$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}}$$

$$X[0] = \sum_{n=0}^{N-1} x[n]\, \cancel{e^{-j\frac{2\pi 0 n}{N}}}^{\,1}$$

$$= \sum_{n=0}^{N-1} x[n]$$

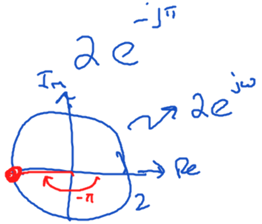
$$= 1 + 2 = 3$$

$$X[1] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi n}{N}}$$

$$= x[0]e^{0} + x[1]\, e^{-j\frac{2\pi}{2}}$$

$$= 1 + \cancel{2e^{-j\pi}}^{\,-2}$$

$$= -1$$



$$X[k] = [3 \quad -1]$$