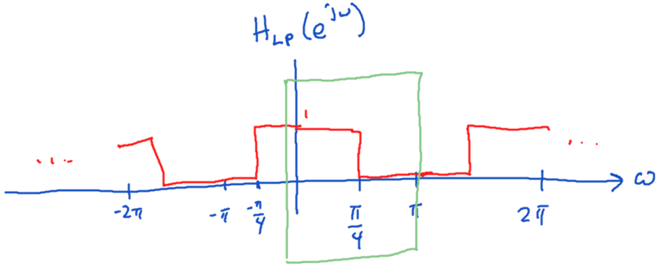
- LPF ✓
- Band limited signals ✓
- DTFT properties ✓
- Energy density spectrum ☆
- DTFT by MATLAB
- Convolution ✓

Ideal Lowpass Filter

$$H_{LP}(e^{j\omega}) = \begin{cases} 1 & 0 \le \omega < \pi/4 \\ 0 & \pi/4 \le \omega < \pi \end{cases}$$



$$h_{LP}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\frac{1}{jn} e^{j\omega n}\Big]_{\omega=-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi jn}\left[e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right]$$

$$= \left(\frac{1}{\pi n}\right)\frac{1}{j2}\left[e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right]$$

θ

$$= \boxed{\frac{1}{\pi n}\sin\left(\frac{\pi}{4}n\right)}$$

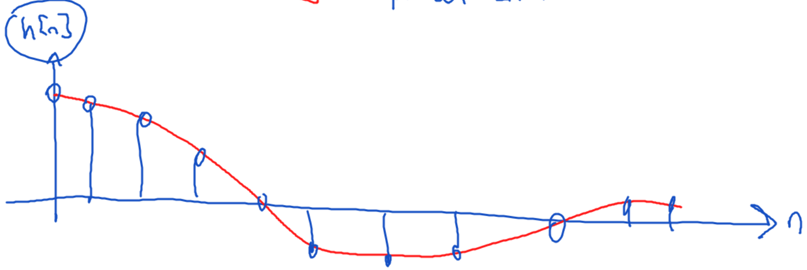
exact impulse response
perfect LPF

Euler's Relations

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\cos(\theta) = \frac{1}{2}\left[e^{j\theta} + e^{-j\theta}\right]$$

$$\sin(\theta) = \frac{1}{j2}\left[e^{j\theta} - e^{-j\theta}\right]$$



$$x[n] \longrightarrow \boxed{h[n]}^{\text{LPF}} \longrightarrow y[n]$$

DTFT

$$X(e^{j\omega}) \longrightarrow \boxed{H(e^{j\omega})} \longrightarrow Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

$x[n]$ $\quad$ DTFT $X(e^{j\omega})$ $\quad$ time shift

$$x[n-3] \Longleftrightarrow e^{-j\omega 3} X(e^{j\omega})$$

$$= \left| e^{-j\omega 3} X(e^{j\omega}) \right|$$

$$= \left| e^{-j\omega 3} \right| \left| X(e^{j\omega}) \right|$$

$$= 1 \cdot \left| X(e^{j\omega}) \right|$$

aside

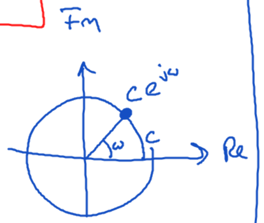
$$|ab| = |a|\,|b|$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$\angle ab = \angle a + \angle b$$

$$\angle \frac{a}{b} = \angle a - \angle b$$

$$c e^{j\omega} = c \angle \omega$$



$$\boxed{x[n] = 5^{-n} u[n-2]}$$

$$= \left(\tfrac{1}{5}\right)^n u[n-2]$$

$$a^n u[n] \Longleftrightarrow \frac{1}{1 - a e^{-j\omega}}$$

$$= \left(\tfrac{1}{5}\right)^2 \left(\tfrac{1}{5}\right)^{n-2} u[n-2] \Longleftrightarrow \left(\tfrac{1}{5}\right)^2 \frac{e^{-j\omega 2}}{1 - \left(\tfrac{1}{5}\right) e^{-j\omega}}$$

$$= \left(\tfrac{1}{5}\right)^2 \left(\tfrac{1}{5}\right)^n \left(\tfrac{1}{5}\right)^{-2} u[n-2]$$

$$= \left(\tfrac{1}{5}\right)^2 \left(\tfrac{1}{5}\right)^n \left(\tfrac{1}{5}\right)^{-2} u[n-2]$$

$$\boxed{\left(\tfrac{1}{5}\right)^2 \cdot \frac{e^{-j\omega 2}}{1 - \left(\tfrac{1}{5}\right) e^{-j\omega}}}$$

DTFT Symmetries

assume real $x[n]$

$$x[n] = x_{even}[n] + x_{odd}[n] \Leftrightarrow X(e^{j\omega}) = X_{Re} + jX_{Im}$$

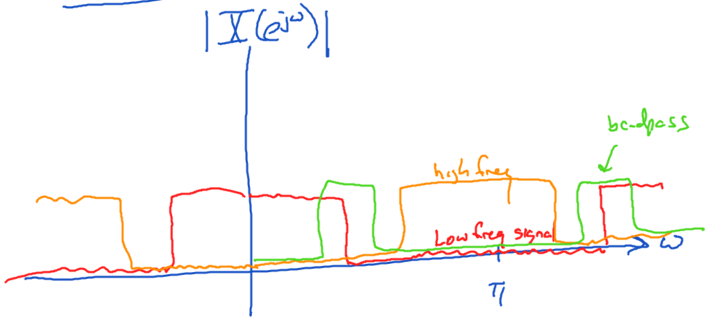
recall

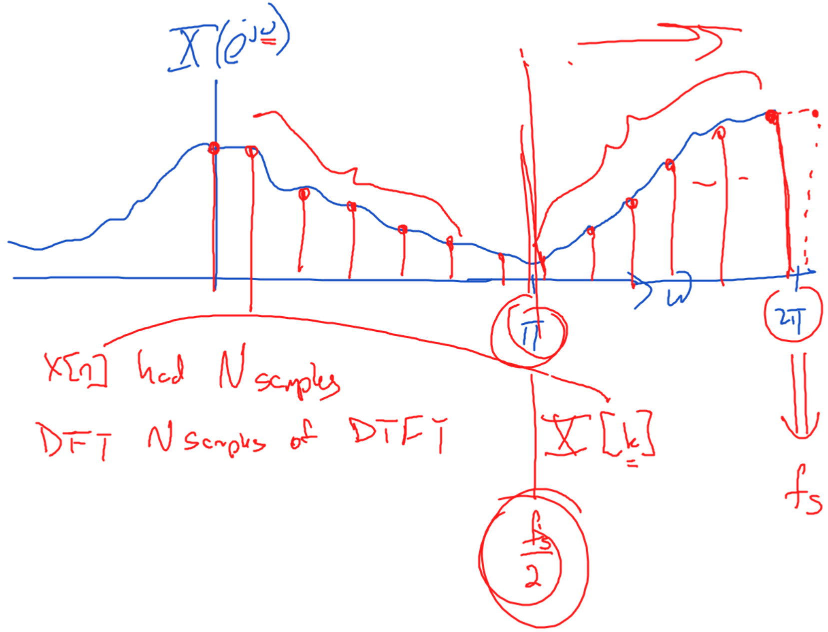
$$x_{even}[n] = \tfrac{1}{2}\left[x[n] + x[-n]\right]$$

$$x_{odd}[n] = \tfrac{1}{2}\left[x[n] - x[-n]\right]$$

$x_{even}[n]$	X_{Re} even around $\omega = 0$
$x_{odd}[n]$	X_{Im} odd "
real $x[n]$ any symm.	$X(e^{j\omega}) = X^*(e^{-j\omega})$ $\lvert X(e^{j\omega})\rvert$ even around $\omega = 0$ $\angle X(e^{j\omega})$ odd around $\omega = 0$

Bandlimited Signals





$x[n]$ had N samples

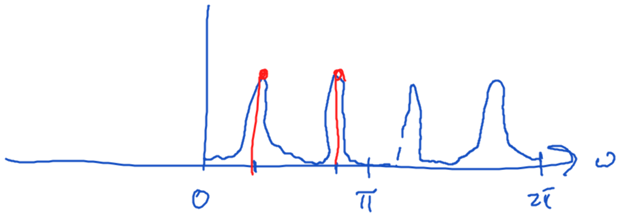
DFT N samples of DTFT

Energy Density Spectrum

What is the energy of $x[n]$ at different frequencies?

ans

$|X(e^{j\omega})|^2$



$$[1 \quad 0 \quad 0 \quad -6]$$

$$a + be^{-j\omega} + ce^{-j2\omega} + de^{-j3\omega}$$

$$[a \quad b \quad c \quad d]$$

DTFT using MATLAB

$\boxed{\text{freqz}}$ if $X(e^{j\omega})$ known already

Ex $$X(e^{j\omega}) = \frac{2+3e^{-j\omega}}{1-6e^{-3j\omega}}$$

```
>> w = linspace(0,pi, 1000)
>> X = freqz([2 3],[1 0 0 -6], w)
```

same $X = (2+3*\exp(-j*w))./(1-6*\exp(-3*j*w))$;

```
>> plot(w, abs(X))
```

or, for energy density spectrum

```
plot(w, abs(X).^2)
```

$x_{sp}[n] = [4 \; -3 \; 6 \; 9 \; 32 \; 2 \; 3 \; -9 \; \ldots]$ $f_s = 44100$ Hz

$|X(e^{j\omega})|^2$