Obj

Freq domain overview

- DTFT - don't get behind
  - Intuition
  - math
  - convergence
  - examples !

Freq Domain Overview

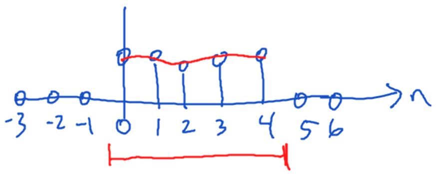
Signals \ freq domain / ch 3

Systems / freq domain \ ch 4

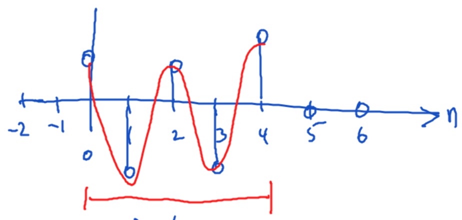
CT	DT
Laplace Transforms any $x(t)\,u(t)$ $\quad X(s) = \int f(t) e^{-st} dt$	Z transform (#3) any $x[n]u[n]$ $\quad X(z)$
Fourier Series periodic $x(t)$ $\quad C_k = \int_{\langle T \rangle} f(t) e^{\frac{-j\omega k t}{T}} dt$	DFT (#2) finite length DTFT
Fourier Transform any $x(t)$ $\quad X(e^{j\omega}) = \int f(t) e^{-j\omega t} dt$	DTFT (#1) any $x[n]$ $\quad X(e^{j\omega}) = \sum x[n] e^{-j\omega n}$ today

DTFT - Intuition

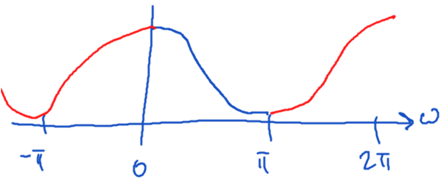
low freq $N=5$



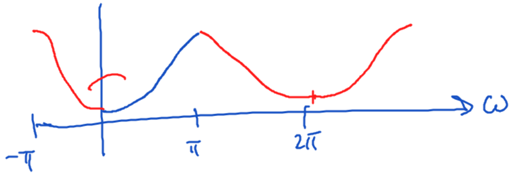
high freq $N=5$



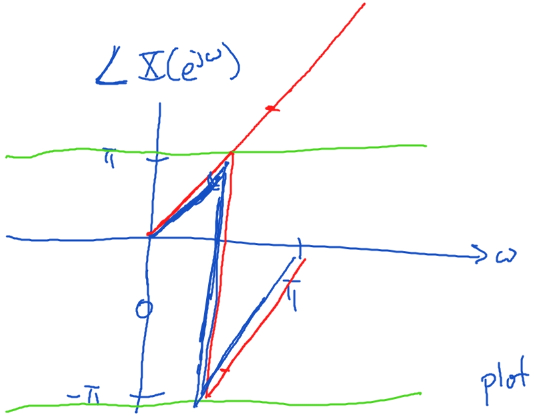
$|X(e^{j\omega})|$



$|X(e^{j\omega})|$



$\angle X(e^{j\omega})$



plot(ω, angle(X)) limited $-\pi, \pi$

plot(ω, unwrap(angle(X)))

DTFT - Math

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

↑ complex ↑ continuous of ω ↑ real, discrete

used

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

never used

DTFT Example

$x[n] = 3\left(\frac{1}{2}\right)^n u[n]$

Find DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} \quad u[n]$$

$$= \sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n e^{-j\omega n}$$

aside

$$\sum_n k \cdot f(n) = k \sum_n f(n)$$

$$\sum_n \left(f(n)+g(n)\right) = \sum_n f(n) + \sum_n g(n)$$

$$a^n b^n = (ab)^n, \quad a^b a^c = a^{b+c}$$

$$= 3 \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n$$

$$X(e^{j\omega}) = 3 \, \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

aside

$$y = \sum_{n=0}^{\infty} a^n = 1 + a^1 + a^2 + a^3 + \dots$$

$$ay = \quad a + a^2 + a^3 + \dots$$

$$y - ay = 1$$

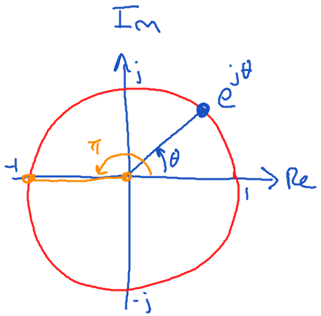
$$y(1-a) = 1 \Rightarrow y = \frac{1}{1-a}$$

Euler's Relations

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left[e^{j\theta} + e^{-j\theta}\right]$$

$$\sin\theta = \frac{1}{j2}\left[e^{j\theta} - e^{-j\theta}\right]$$



Ex $e^{j\pi} = -1$

$e^{j\frac{\pi}{2}} = j$

Ex $\left(\frac{1}{j2} e^{j(\frac{\pi}{4}n)} - e^{-j(\frac{\pi}{4}n)}\right) j2$

$= j2\sin\left(\frac{\pi}{4}n\right)$