Sampling

1. Let $y(t) = \cos(\overset{\omega_1}{6\pi} t) + 2\cos(\overset{\omega_2}{14\pi} t) - 5\cos(\overset{\omega_3}{26\pi} t)$

   a) find the minimum sampling frequency to prevent aliasing

Nyquist rate sample $2\times$ max freq in CT samples/sec

$f_1 = \frac{\omega_1}{2\pi} = \frac{6\pi}{2\pi} = 3\text{Hz}$ $\quad f_3 = \frac{\omega_3}{2\pi} = \frac{26\pi}{2\pi} = 13\text{Hz}$ $\quad f_s = 2 f_{max} = 26\text{Hz} = \boxed{26 \frac{\text{samples}}{\text{sec}}}$

   b) find y[n] if sampled at 10Hz. Keep all discrete frequencies between 0 and π rads/sec.

$$\underset{DT}{y[n]} = \underset{CT}{y(t = nT_s)} \qquad T_s = \frac{1}{f_s} = \frac{1}{10} \text{ sec/sample}$$

$$= \cos\left(6\pi n \tfrac{1}{10}\right) + 2\cos\left(14\pi n \tfrac{1}{10}\right) - 5\cos\left(26\pi n \tfrac{1}{10}\right)$$

$$= \cos(\underbrace{0.6\pi}_{\omega_1} n) + 2\cos(\underbrace{1.4\pi}_{\omega_2} n) - 5\cos(\underbrace{2.6\pi}_{\omega_3} n)$$

$$= \cos(0.6\pi n) + 2\cos(1.4\pi n - 2\pi n) - 5\cos(2.6\pi n - 2\pi n)$$

$$= \cos(0.6\pi n) + 2\cos(-0.6\pi n) - 5\cos(0.6\pi n)$$

$$= \cos(0.6\pi n) + 2\cos(0.6\pi n) - 5\cos(0.6\pi n)$$

$$\boxed{\cos(x) = \cos(-x)} \qquad = \boxed{-2\cos(0.6\pi n)}$$

$$\cos(1.7\pi n) = \cos(1.7\pi n - 2\pi n)$$
$$= \cos(-0.3\pi n)$$
$$= \cos(0.3\pi n) \qquad \omega = 0.3\pi$$

$$\sin(1.7\pi n) = \cos(1.7\pi n - \pi/2)$$
$$= \cos(1.7\pi n - 2\pi n - \pi/2)$$
$$= \cos(-0.3\pi n - \pi/2)$$
$$= \cos(0.3\pi n + \pi/2)$$

$+3644n\pi$

DT CT

2. Let $y[n] = 2\cos(\boxed{\frac{1}{2}}\pi n)$. What was the original signal if f_s = 7Hz assuming there was no aliasing?

$t = nT_s$

$n = t/T_s$

$f_s = 7$ samples/sec

$T_s = 1/7$ sec/sample

$$\underset{DT}{y[n]} = \underset{CT}{y(t = nT_s)}$$

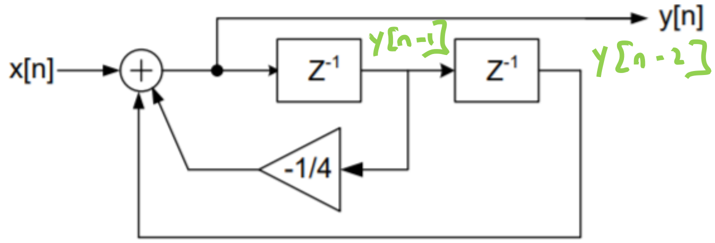
$$\underset{CT}{y(t)} = \underset{DT}{y[t/T_s]}$$

$$= 2\cos\left(\tfrac{1}{2}\pi \, t/_{1/7}\right)$$

$$= \boxed{2\cos\left(\tfrac{7\pi}{2} t\right)}$$

Block Diagram ↔ DE

3. Given the system on the right



a) Find the DE

b) Let $x[n] = n^2u[n]$. Plot $y[n]$ for $0 \le n \le 2$

Block Diagram → DE difference eqn

1. Give signal line name (eg $w[n]$) before every delay and then name it correspondingly after delay ($w[n-1]$)
2. Write eqn around each summer (at the output) ⊕
3. Put in standard form $y[n]$ left, $x[n]$ right

- $y[n] = x[n] + y[n-2] - \frac{1}{4}y[n-1]$
- $\boxed{y[n] + \frac{1}{4}y[n-1] - y[n-2] = x[n]}$

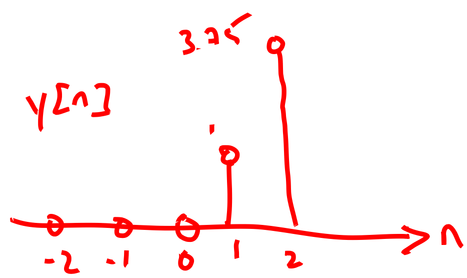
$x[n] = [\ldots 0 \ 0 \ 0 \ \underset{\uparrow}{0} \ 1 \ 4 \ldots]$

$y[n] = [\ldots 0 \ 0 \ \cancel{0} \ \boxed{\underset{\uparrow}{0} \ 1 \ 3.75}$

$y[0] = x[0] + y[-2] - \frac{1}{4}y[-1]$
$= 0 + 0 - \frac{1}{4}0$
$= 0$

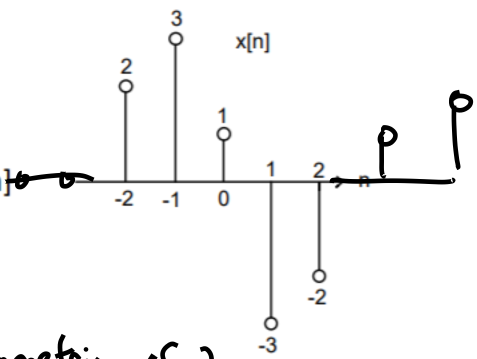
$y[1] = x[1] + y[-1] - \frac{1}{4}y[0]$
$= 1 + 0 - \frac{1}{4} \cdot 0$
$= 1$

$y[2] = x[2] + y[0] - \frac{1}{4}y[1]$
$= 4 + 0 - \frac{1}{4} = 3\frac{3}{4}$



Symmetry

4. Find the ca part of x[n]



conjugate antisymmetric x[n]

$$x_{ca}[n] = \frac{1}{2}\left\{ x[n] - x^*[-n] \right\}$$

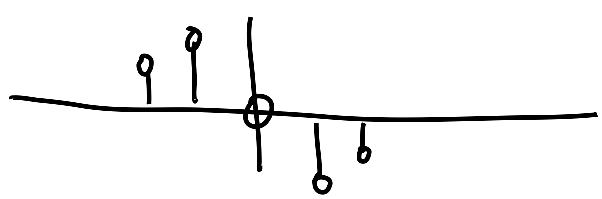
$$= \frac{1}{2}\left\{ [2\ \ 3\ \ \underset{\uparrow}{1}\ \ -3\ \ -2] - [-2\ \ -3\ \ \underset{\uparrow}{1}\ \ 3\ \ 2]^* \right\}$$

$$= \frac{1}{2}\left\{ [2\ \ 3\ \ \underset{\uparrow}{1}\ \ -3\ \ -2] - [-2\ \ -3\ \ \underset{\uparrow}{1}\ \ 3\ \ 2] \right\}$$

$$= \frac{1}{2}\left\{ [4\ \ 6\ \ \underset{\uparrow}{0}\ \ -6\ \ -4] \right\}$$

$$= [2\ \ 3\ \ \underset{\uparrow}{0}\ \ -3\ \ -2]$$

ca



5. Find k to make the following finite-length sequence w[n] periodic conjugate symmetric:

$w[n] = [\,\underset{\uparrow}{9} \quad 3-j \quad -1 \quad k\,]$

$9 \quad 3-j \quad -1 \quad k \quad 9 \quad 3-j \quad -1 \quad k \quad \underset{\uparrow}{9} \quad 3-j \quad -1 \quad k \quad 9 \quad 3-j \quad -1 \quad k$

$*$

$*$

$*$

$k = (3-j)^*$

$\;\;= 3+j$

$(3-j) = k^*$

$\boxed{k = 3+j}$ ✓

check

$w[n] = [\,\underset{\uparrow}{9} \quad 3-j \quad -1 \quad 3+j\,]$

$W_{cs}[n] = \frac{1}{2}\{ w[\langle n \rangle_N] + w^*[\langle -n \rangle_N] \}$ $\qquad N=4$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0 \ldots 3$

$n=0$

$W_{cs}[0] = \frac{1}{2}\{ w[\langle 0 \rangle_4 \; + \; w^*[\langle 0 \rangle_4 \}$

$\frac{1}{2}\{ w[0] + w^*[0] \} = \frac{1}{2}\{9+9\} = 9$ ✓

$W_{cs}[1] = \frac{1}{2}\{ w[\langle 1 \rangle_4] + w^*[\langle -1 \rangle_4] \}$

$= \frac{1}{2}\{ w[1] + w^*[3] \}$

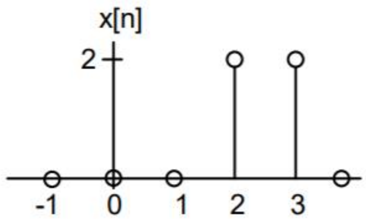
$= \frac{1}{2}\{ (3-j) + (3-j) \} = \frac{1}{2}(6-j2) = 3-j$ ✓

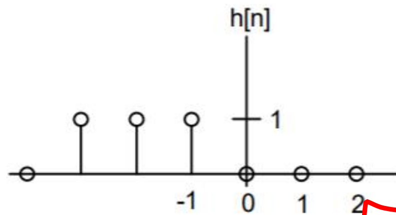
$\vdots$

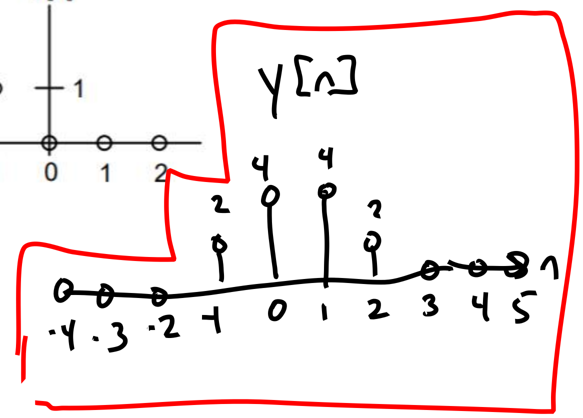
$W_{cs}[n] = w[n]$ ✓

Convolution

6. Given the following x[n] and h[n], graphically convolve to find y[n] = x[n] * h[n]. Both signals are zero outside the graphed regions.

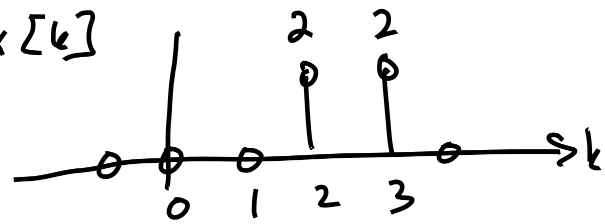






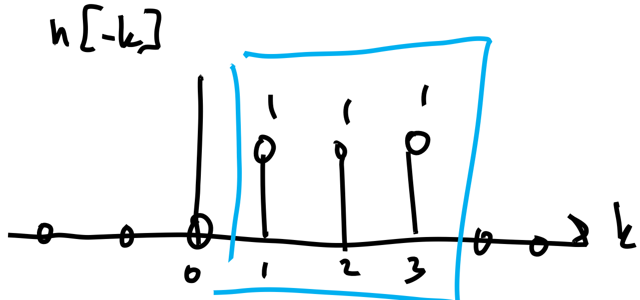
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

x[k]

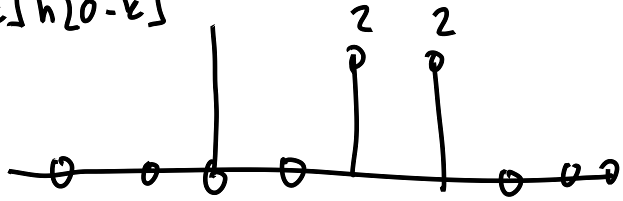


y[0]
n = 0

h[-k]

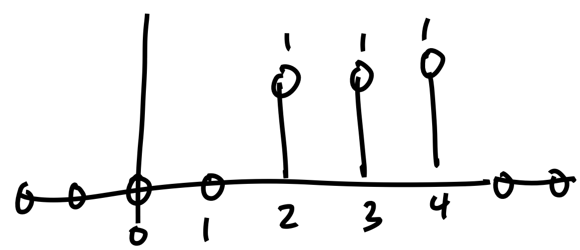


x[k] h[0-k]

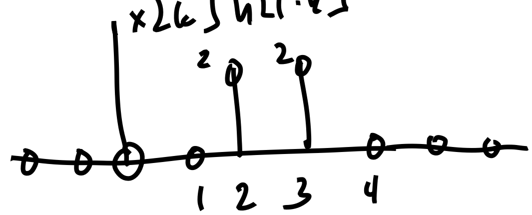


$y[0] = 4$ $y[-1] = 2$

y[1]   h[1-k]



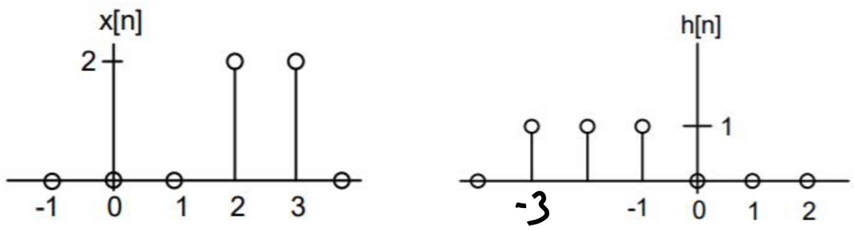
x[k] h[1-k]



$y[1] = 4$ $y[2] = 2$ $y[3] = 0$

Convolution

6. Given the following x[n] and h[n], graphically convolve to find y[n] = x[n] * h[n]. Both signals are zero outside the graphed regions.



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]\, h[-k]$$

$$= \sum_{k=-3}^{3} x[k]\, h[-k]$$

$$= \overset{k=-3}{x[-3]h[3]} + \overset{k=-2}{x[-2]h[2]} + \overset{k=-1}{x[-1]h[1]} + \overset{k=0}{x[0]h[0]} + \overset{k=1}{x[1]h[-1]}$$

$$+ x[2]\, h[-2] + \overset{k=3}{x[3]\, h[-3]}$$

$$= \overset{k=-3}{0} + \overset{k=-2}{0} + \overset{k=-1}{0} + \overset{k=0}{0} + \overset{k=1}{0} + \overset{k=2}{(2)(1)} + \overset{k=3}{(2)(1)}$$

$$= 4$$

$$y[1] = \sum x[k]\, h[1-k]$$

$$y[2] = \ldots$$