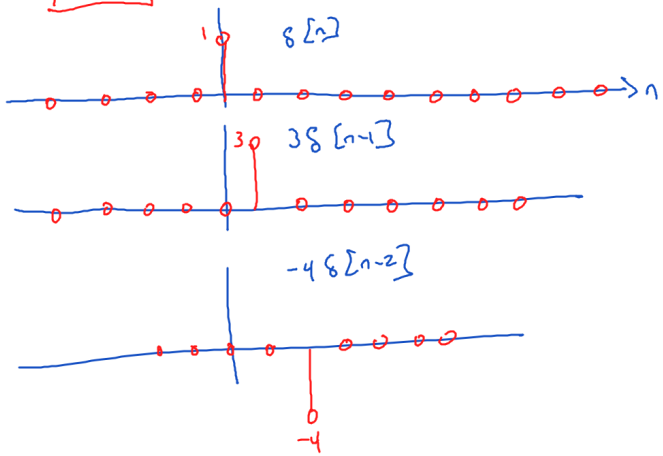
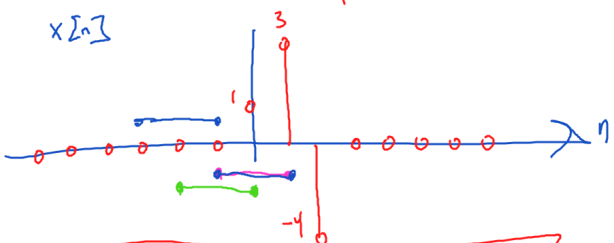
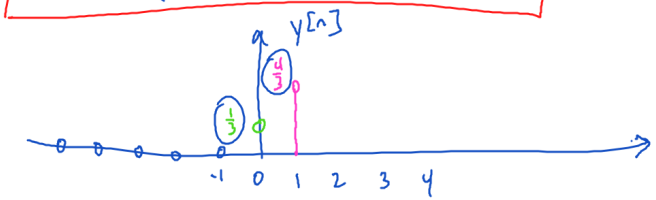
$x[n] = \delta[n] + 3\delta[n-1] - 4\delta[n-2]$

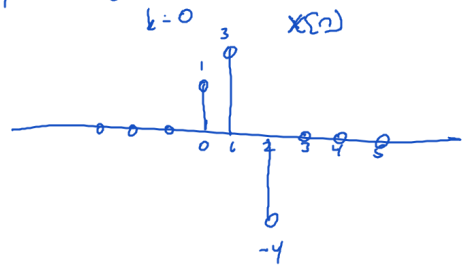




$$y[n] = \left(x[n] + x[n-1] + x[n-2]\right)\frac{1}{3}$$



$$y[n] = \frac{1}{3}\sum_{k=0}^{2} x[n-k]$$



y[n]

$y[0] = (x[0] + x[-1] + x[-2])/3$

$= (1 + 0 + 0)/3 = 1/3$

$y[1] = (x[1] + x[0] + x[-1])/3$

$(3 + 1 + 0)/3 = 4/3$

Obj Convolution
- by math ✓
- by properties ✓
- by MATLAB
- graphically

Convolution

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

$x[n]$

$x[n] = \delta[n] + 2\delta[n-1] + \delta[n-2] \rightarrow y[n] = h[n] + 2h[n-1] + h[n-2]$

CONVOLUTION

In general

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]\,h[k]$$

$$= x[n] * h[n]$$

$$y(t): \int_{\tau=-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

Gotcha:

operation	rest of world	Mitra
convolution	$*$	$\circledast$
Circular convolution	$\circledast$	$\textcircled{N}$
Conjugation	$(2\angle 3^\circ)^* = 2\angle -3^\circ$ $(2+j)^* = 2-j$	$*$

Convolution Properties

Common: $y = x[n] * h[n] = h[n] * x[n]$

$y = x * h = h * x$

assoc: $y = (x * h_1) * h_2 = x * (h_1 * h_2)$

$x[n] \rightarrow \boxed{h_1} \rightarrow \boxed{h_2} \rightarrow y[n]$

$x[n] \rightarrow \boxed{h_1 * h_2} \rightarrow y[n]$

distrib $y = x * (h_1 + h_2) = x * h_1 + x * h_2$

$x[n] \rightarrow \boxed{h_1[n] + h_2[n]} \rightarrow y[n]$

Convolution by MATLAB

good news: `conv` bad news: no indexing

magnitude	indexing
eg >> x = [1 2 2 1]	nx = 0:3
>> h = [1 2]	nh = 0:1
>> y = conv(x, h)	ny = 0:4
= [1 4 6 5 2]	stem(ny, y)

-1:2

How do you create ny?

Rule 1: if lengths N_x, N_h (eg nx = 0: N_x-1 or = 1: N_x)

then $N_y = N_x + N_h - 1$

Rule 2 if x and h begin at nx_1 and nh_1

then y begin $nx_1 + nh_1$

eg $x = [1\ 2\ 2\ 1]$ (n = -1, 0, 1, 2)

$h = [1\ 2]$

$y = conv(x, h)$

$= [1\ 4\ 6\ 5\ 2]$

nx = -1:2

nh = [-1 0]

ny = -2:2

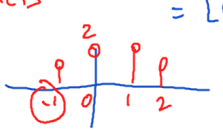
$N_y = N_x + N_h - 1 = 4 + 2 - 1 = 5$

$n_{y1} = n_{x1} + n_{h1} = (-1) + (-1) = -2$

$ny_1 : ny_1 + N_y - 1$

stem(ny, y)

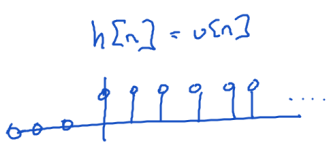
x[n]

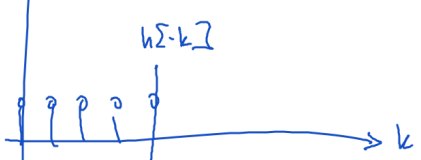


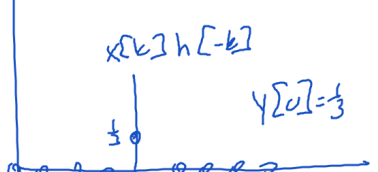
Graphical

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

ex y[0] n=0







y[1] n=1