aliasing

- max freq of CT signal $> 2 f_{sample}$
- $0 \le \omega \le \pi$

- $\cos(1.4\pi n)$ $\quad \omega = 1.4\pi$

$= \cos(1.4\pi n - 2\pi n)$ equivalent freq

$= \cos(-0.6\pi n)$

$= \cos(0.6\pi n)$ $\quad \omega = 0.6\pi$

aliasing

$\cos(1.4\pi t)$

$\cos(0.6\pi t)$

if aliasing at 2π

$x[n] = \cos(6\pi n)$ $\quad \omega = 6\pi$

$= \cos(6\pi n - 6\pi n)$

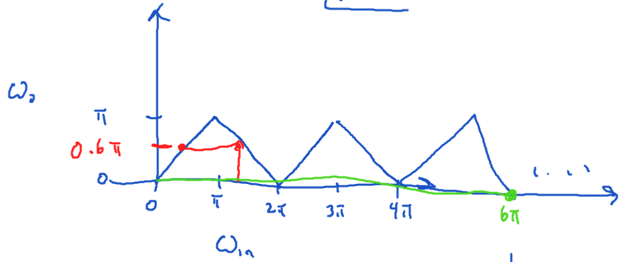
$= \cos(0)$

$= 1$

$\omega = 0$ radians/sample

$\boxed{\cos(\omega n)}$

$\cos(\cancel{2\pi f}\, n)$



CT

$\boxed{\begin{array}{l}\cos(\omega t) \\ \cos(2\pi f t)\end{array}}$

ω [rad/sec] $\quad t$ [sec]

f $[Hz = \frac{1}{sec}]$ $\quad t$ [sec]

$f_s = \frac{samples}{sec}$ $\quad$ 3Hz

$\boxed{\text{every sec 3 samples}}$

Qu. 2

$x(t) = 4\cos(10t)$

Find the min f_s to prevent aliasing

CT	DT
Ω_0 rad/sec	ω_0
f_0 Hz = 1/sec	

① $x(t) = 4\cos(\Omega_0 t)$

or $4\cos(2\pi f_0 t)$

$\Omega_0 = 10 = 2\pi f_0 \Rightarrow f_0 = \frac{10}{2\pi} = \frac{5}{\pi}$ Hz

② $\min f_s = (2)(f_{\text{max in signal}})$

$= 2\left(\frac{5}{\pi}\right)$ Hz

$= \frac{10}{\pi} \approx 3.18$ Hz

Every sec, need $\geq \boxed{3.18}$ samples to prevent aliasing

DT Systems — Represent / Characterize

DT Systems Representation

$$x[n] \to \boxed{h[n]} \to y[n]$$

1. Difference Eqn (may or may not be unique)

eg Accumulator

$$y[n] = x[-\infty] + \cdots x[n-2] + x[n-1] + x[n]$$

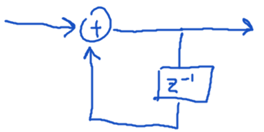
$$= \sum_{k=-\infty}^{n} x[k]$$

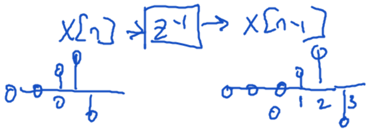
$$= \sum_{k=-\infty}^{n-1} x[k] + x[n]$$

$$y[n] = y[n-1] + x[n]$$

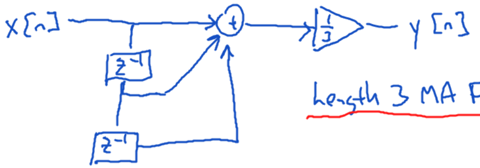
$$y[n] - y[n-1] = x[n]$$

2. Block diagrams

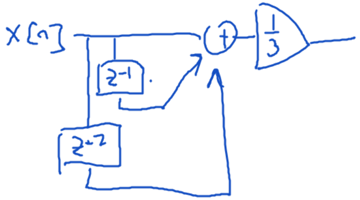




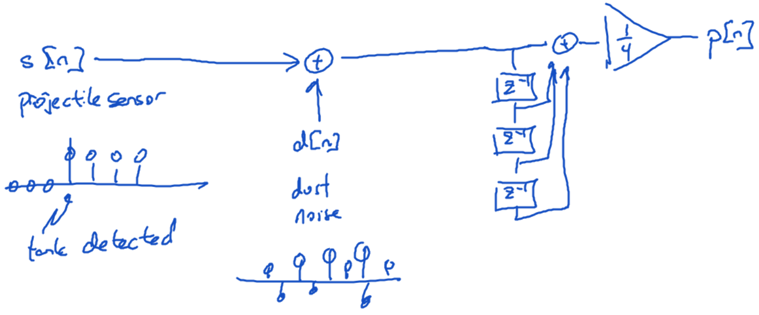
ex Moving average filter $y[n] = \dfrac{x[n-2] + x[n-1] + x[n]}{3}$



Length 3 MA Filter



Real-World Ex



Characterization of Systems

- **Linear**
  1. Scaling $A\,x[n] \longrightarrow \boxed{h[n]} \longrightarrow A\,y[n]$
  2. Superposition if $\begin{cases} x_1 \longrightarrow y_1 \\ x_2 \longrightarrow y_2 \end{cases}$

     then $x_1 + x_2 \longrightarrow y_1 + y_2$

- **Time Invariant**

  ex: HPF today, HPF tomorrow

  $h[n]$ does not change

  vs.

  $h[n, n_1]$

  particular time using it

Linear, Time-Invariant = LTI

Causality $y[n]$ only depends on current or past $x[n]$

$h[n]$ is 0 for $n < 0$

BIBO Stability bounded inputs → bounded output

$\sum |h[n]| < \infty \Rightarrow$ BIBO stable

$y[n] = x[n] + y[n-1]$ if $x[n] = u[n]$

output grows

Lossless if output energy = input energy

energy $\mathcal{E}_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$

Passive

if output energy $\leq$ input energy