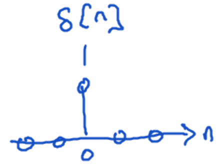
Obj

Common Sequences
- unit sample
- unit step
- sinusoid
- sequences $\Leftrightarrow$ unit sample
- complex exponential

Sampling
- definition
- DT limited freq range
- Sampling Theorem

Unit Sample

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{ow} \end{cases}$$



Ex $\delta[-6] = 0$

$\delta[n-1]$

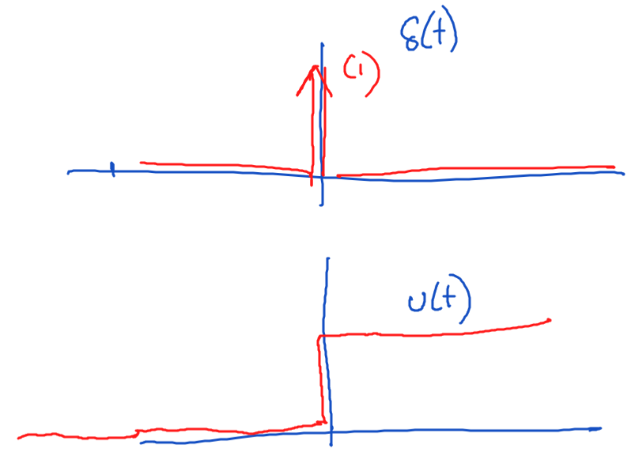
Unit Step

$$u[n] = \begin{cases} 0 & n<0 \\ 1 & n \geq 0 \end{cases}$$

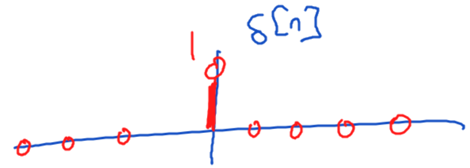
$$\sum_{k=-\infty}^{n} \delta[k] = u[n] \quad \text{DT}$$

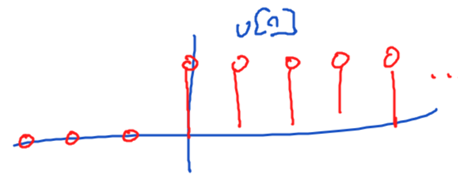
$$\int_{\tau=-\infty}^{t} \delta(t)\, d\tau = u(t) \quad \text{CT}$$

CT



$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$





Sinusoids

discrete frequency

phase

$$x[n] = A\cos(\omega_0 n + \phi) \quad \text{DT} \quad \text{unique } 0 \leq \omega_0 \leq \pi$$

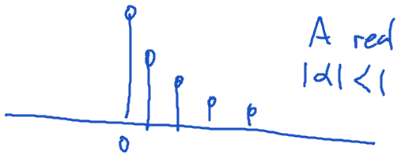
$$x(t) = A\cos(\Omega_0 t + \phi) \quad \text{CT} \quad 0 \leq \Omega_0 < \infty$$

Continuous time frequency

Complex Exponential

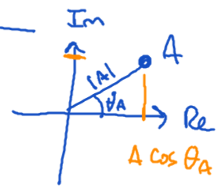
- $\boxed{x[n] = A\,\alpha^n}$ real or complex

A real
$|\alpha| > 1$



A real
$|\alpha| < 1$

- $A = |A| e^{j\theta_A} = A\angle\theta_A =$

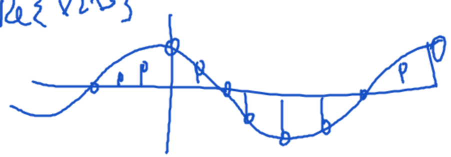


$A\cos\theta_A + jA\sin\theta_A$

- $\alpha = |\alpha| e^{j\theta_\alpha}$

$$x[n] = |A| e^{j\theta_A} \left(|\alpha| e^{j\theta_\alpha}\right)^n$$
$$= |A||\alpha|^n e^{j(\theta_A + \theta_\alpha n)}$$
$$= |A||\alpha|^n \cos(n\theta_\alpha + \theta_A) + j\,|A||\alpha|^n \sin(n\theta_\alpha + \theta_A)$$

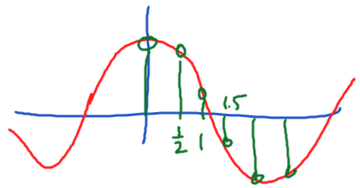
$Re\{x[n]\}$ $|\alpha| = 1$



Sampling

Def

$x(t)$ CT $\longrightarrow$ [Sample every T_s seconds] $\longrightarrow$ $\boxed{x[n] = x(nT_s)}$ DT



$T_s = 0.5$ s

$f_s = \frac{1}{T_s} = 2$ Hz

Limited Freq Range

CT : $x(t) = \cos(\Omega_0 t)$ $0 \le \Omega_0 < \infty$

DT : $x[n] = \cos(\omega_0 n)$ $0 \le \omega_0 \le \pi$ unique

Ex

$\omega_0 = 0$: $x[n] = \cos(0 \cdot n)$ $[1\ 1\ 1\ 1 \dots]$ lowest freq $\omega_0 = 0$

$\omega_0 = \pi$: $x[n] = \cos(\pi \cdot n)$ $[1\ -1\ 1\ -1\ 1 \dots]$ highest freq $\omega_0 = \pi$

$\omega_0 = 2\pi$: $x[n] = \cos(2\pi n)$ $[1\ 1\ 1\ 1\ 1 \dots]$ $\omega_0 = 2\pi$

same!

Ex

$$\boxed{\begin{aligned} x[n] &= x(nT_s) \\ T_s &= \frac{1}{f_s} \end{aligned}}$$

$\underline{0 \le \omega_0 \le \pi}$

Ex

$y_1[n] = \cos(\underline{0.6\pi}\, n)$ $\qquad \omega_1 = 0.6\pi$

$y_2[n] = \cos(\underline{1.4\pi}\, n)$ $\qquad \omega_2 = 1.4\pi$

$= \cos((2\pi - 0.6\pi)n)$

$= \cos(2\pi n - 0.6\pi n)$ $\qquad$ add or sub any multiple of 2π

$= \cos(-0.6\pi n)$ $\qquad$ but $\cos(-x) = \cos(x)$

$= \cos(\underline{0.6}\pi n)$ $\qquad \omega_2 \equiv 0.6\pi$

$= y_1[n]$!

$y_3[n] = \cos(2.6\pi n)$ $\qquad \omega_3 = 2.6\pi$

$= \cos((2\pi + 0.6\pi)n)$

$= \cos(2\pi n + 0.6\pi n)$

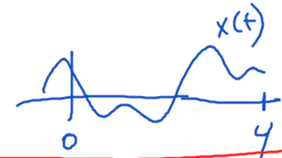
$= \cos(\underline{0.6}\pi n)$ $\qquad \omega_3 = \underline{0.6\pi}$

$= y_1[n]$!

Aliasing!

$0 \le \omega_0 \le \pi$

Sampling Theorem



Can perfectly capture all signal information if $f_s \ge \underbrace{2 \cdot f_{\text{max freq of CT signal}}}_{\text{Nyquist Frequency}}$

Otherwise Aliasing

high freq $\Rightarrow$ low freq.

$x(t) \rightarrow$ [analog anti-aliasing filter] $\rightarrow$ [Sample $f_s = 40$ Hz] $\rightarrow x[n]$

eg 20 Hz