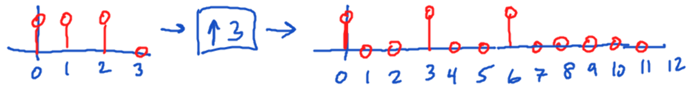
Obj

- Up / down sampling
- Symmetry (odd, even, conjugate, periodic)
- Periodic vs aperiodic waveforms
- Energy vs. power waveforms
- Absolutely summable or square summable

Up Sampling

does not change $x[0]$

$$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow y[n]$$

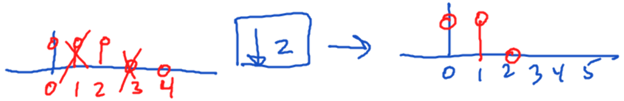


Length 4 Length 12

$$y[n] = \begin{cases} x\left[\frac{n}{L}\right], & \text{if } n/L \text{ is integer} \\ 0, & \text{ow} \end{cases}$$

Down sampling

$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow y[n]$$



Question

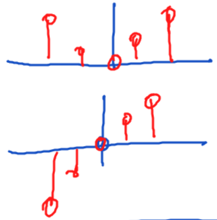
how to upsample by 1.5?

$$\longrightarrow \boxed{\uparrow 3} \longrightarrow \boxed{\downarrow 2} \longrightarrow$$

filter

Symmetry

Recall $\begin{cases} \text{even} \\ \text{odd} \end{cases}$



$x[n] = x[-n]$ flip around y

$x[n] = -x[-n]$ flip around y, x

$x[n] = x_o[n] + x_e[n]$

	"even"	"odd"
real + ∞ length $x[n] = x_o[n] + x_e[n]$	$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\}$ eg $x[n] = n^3$ $x_e[n] = \frac{1}{2}\{n^3 + (-n)^3\}$ $= 0$	$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\}$ eg $[\ldots -1\ -1\ -1\ \underset{\uparrow}{0}\ 1\ 1\ 1 \ldots]$
Complex, ∞ length $x[n] = x_{cs}[n] + x_{ca}[n]$	conjugate symmetric $x_{cs}[n] = \frac{1}{2}\{x[n] + x^*[-n]\}$ means conjugate $(2+j6)^* = 2-j6$ $(3\angle 20°)^* = 3\angle -20°$ eg $[\ldots 0\ 0\ 1+j\ \underset{\uparrow}{7}\ 1-j\ 0\ 0 \ldots]$	conjugate anti symmetric $x_{ca}[n] = \frac{1}{2}\{x[n] - x^*[-n]\}$ $[\ldots 0\ 1-j\ \underset{\uparrow}{0}\ -1-j\ 0 \ldots]$
complex, finite length $0 \le n < N$ $x[n] = x_{pcs}[n] + x_{pca}[n]$	periodic conj sym $x_{pcs}[n] = \frac{1}{2}\{x[n] + x^*[\langle -n \rangle_N]\}$	periodic conj anti $x_{pca}[n] = \frac{1}{2}\{x[n] - x^*[-\langle n \rangle_N]\}$

Math

- $\{\underset{\uparrow}{x[0]}\ x[1]\ x[2]\}$ think $\{\overset{x[-2]}{x[1]}\ \overset{x[-1]}{x[2]}\ \underset{\uparrow}{x[0]}\ x[1]\ x[2]\}$
- instead of $x[-n]$, now $x[\langle -n \rangle_N]$

where $\langle n \rangle_N$ "n modulo N" means $n \pm$ many N's to bring it into range of $0 \le n < N$

eg $\langle 3 \rangle_6 = 3$, $\langle 12 \rangle_6 = 0$, $\langle 11 \rangle_6 = 5$, $\langle 0 \rangle_6 = 0$

$\langle -1 \rangle_6 = 5$

Ex Find periodic conj. sym. part of $[\underset{\uparrow}{0}\ j\ 2]$

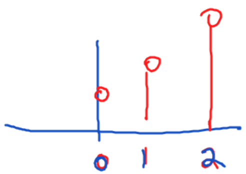
$x_{pcs}[n] = \frac{1}{2}\{x[n] + x^*[\langle -n \rangle_N]\}$ think $\cdots 0\ j\ 2\ \underset{\uparrow}{0}\ j\ 2\ 0 \ldots$

$= \frac{1}{2}\{[0\ j\ 2] + \{\overset{x[0]^*}{x[\langle -0 \rangle_3]} + \overset{x[2]^*}{x[\langle -1 \rangle_3]} + \overset{x[1]^*}{x[\langle -2 \rangle_3]}\}\}$

$= \frac{1}{2}\{[\underset{\uparrow}{0}\ j\ 2] + [0\ 2\ -j]\}$

$x_{pcs} = [0\ \frac{j+2}{2}\ \frac{2-j}{2}]$ $+\ x_{pca}[n] = x[n]$

$\ldots 0\ 1+\frac{j}{2}\ 1-\frac{j}{2}\ \underset{\uparrow}{0}\ 1+\frac{j}{2}\ 1-\frac{j}{2}\ 0\ 1+\frac{j}{2}\ 1-\frac{j}{2} \ldots$

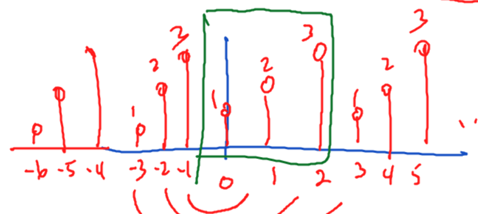


$0 \le n < 3$

finite length $0 \le n < 3$

$\neq \underbrace{x_{cs}[n] + x_{ca}[n]}$

$\neq x_e[n] + x_o[n]$

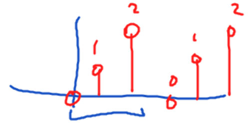


$\Rightarrow x_{pcs}[n] = [\underset{\uparrow}{1}\ 2.5\ 2.5]$

Periodic vs aperiodic

- Notion $\tilde{x}$ Ex

- Formula $\tilde{x}[n] = \tilde{x}[n + kN]$ k any integer, N period



$P = \frac{0+1+4}{3} = \frac{5}{3}$

Energy vs Power Signals

- Energy of $x[n] = \mathcal{E}_x \equiv \sum_{n=-\infty}^{\infty} x^2[n]$
- Power of $x[n] = P_x$ = average energy per sample $= \lim_{N\to\infty} \left(\frac{\text{energy in } N \text{ samples}}{N} \right)$

$$= \lim_{K\to\infty} \frac{1}{2K+1} \sum_{n=-K}^{K} x^2[n]$$

- Ex Energy: $4+1+4+1+\ldots = \infty$

  power $\frac{4+1}{2} = \frac{5}{2}$ $\frac{4+1+4+1}{4} = \frac{5}{2}$

- Power signal if P_x finite $\Rightarrow E_x = \infty$
- Energy signal if $P_x = 0 \Rightarrow E_x$ is finite

- Absolutely summable $= \sum |x[n]| < \infty$
- Square summable $= \sum x^2[n] < \infty$