

1) Find $v(t)$

7-2u(t) $\frac{1}{6}$ F $\frac{1}{6}$ H $t=0$ $2A$

① $t < 0$ $7V$ $i_L = 2A$ $v_L = 7V$ by inspection

② $t = 0^+$ $5V$ 2 $7V$ $v'_L = \frac{1}{L} i_L = 6(2) = 12V/s$
 $v_L(0^+) = 7V$
 $v'_L(0^+) = 12V/s$

③ $t > 0$ $5V$ 2 $\frac{1}{6}$ F $\frac{1}{6}$ H $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1/36}} = 6$
 $\alpha = \frac{R}{2L} = \frac{2}{2/6} = 6$
 $S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -6 \pm 0 \Rightarrow v_n(t) = C_1 e^{-6t} + C_2 t e^{-6t}$

④ $t' = \infty$ $5V$ 2 $v = 5V$ by inspection $\Rightarrow v_f(t) = 5$

⑤ $v(t) = v_n + v_f = C_1 e^{-6t} + C_2 t e^{-6t} + 5$
 $v(0) = C_1 + 5 = 7 \Rightarrow C_1 = 2$
 $v'(t) = -6C_1 e^{-6t} + C_2 e^{-6t} + C_2 + (-6) e^{-6t}$
 $v'(0) = -6C_1 + C_2 = 12$
 $-12 + C_2 = 12 \Rightarrow C_2 = 24$
 $v(t) = 2e^{-6t} + 24te^{-6t} + 5$

2) Find $v(t)$. Similar to above, but solutions look quite different.

7-2u(t) $\frac{1}{13}$ F $1H$ $t=0$ $23/13$ A

$t < 0$ $7V$ 4 $1H$ $i_L = -23/13$ A

$t = 0^+$ $5V$ 4 $1H$ $i_L = -23/13$ A $v'_L = \frac{1}{L} i_L = 13(-23/13) = -23V/s$
 $v_L(0^+) = 7V$
 $v'_L(0^+) = -23V/s$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1/13}} = \sqrt{13}$
 $\alpha = \frac{R}{2L} = \frac{4}{2} = 2$
 $S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2 \pm \sqrt{4 - 13} = -2 \pm j3 \Rightarrow \text{underdamped}$
 $v_n(t) = e^{-2t} [C_1 \cos(3t) + C_2 \sin(3t)]$

$t > 0$ $5V$ 4 $1H$ $\frac{1}{13}$ F

$t = \infty$ $5V$ 4 $v_f = v_\infty = 5V$

$v(t) = v_f + v_n = e^{-2t} [C_1 \cos(3t) + C_2 \sin(3t)] + 5$
 $v(0) = 7 = C_1 + 5 \Rightarrow C_1 = 2$
 $v(t) = e^{-2t} [2 \cos(3t) + C_2 \sin(3t)] + 5$
 $v'(t) = [e^{-2t} [-6 \sin(3t) + 3C_2 \cos(3t)] - 2e^{-2t} [2 \cos(3t) + C_2 \sin(3t)]]$
 $v'(0) = 3C_2 - 2 \cdot 2 = -23 \Rightarrow 3C_2 = -19 \Rightarrow C_2 = -19/3$
 $v(t) = e^{-2t} [2 \cos(3t) - \frac{19}{3} \sin(3t)] + 5V$